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COMMUNICATION: Equivalence of Naylor-Backer Equation and Weller-Steiner Equation in Barrier Separation

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COMMUNICATION

Equivalence of Naylor-Backer Equation and Weller-Steiner Equation in Barrier Separation

Separation of gases or vapors by barriers (both porous and nonporous membranes) results from the preferential permeation of the light component out of a gaseous mixture. Calculations of enrichment in a single stage were first presented by Weller and Steiner (*1*) for nonporous membranes. Also, similar calculations were reported by Naylor and Backer (*2*) for a porous membrane. However, these results can be applied interchangeably to porous and nonporous membranes if a proper separation factor is used. The mathematical forms of these two approaches appear to be entirely different. Therefore, the aim of the present paper is to show that Eq. (15) in Weller and Steiner's paper (*1*) reduces to Eq. (24) of Naylor and Backer's paper (*2*) when the pressure of the undiffused stream becomes zero. These equations permit one to calculate the concentration of the diffused stream when the concentration of the undiffused stream, the separation factor, and the "cut" are known quantities. Both equations are derived based on the nonmixing model.

Equation (15) in Weller and Steiner's paper (*1*) is

$$\frac{n_A}{n_A{}^0} = \left(\frac{n_B}{n_B{}^0}\right)^{\alpha} \tag{1}$$

In order to convert this into an equation that contains x, y, and θ, the following relations are specified:

$$x^0 = \frac{n_A{}^0}{n_A{}^0 + n_B{}^0} = \frac{n_A{}^0}{n^0} \tag{2}$$

$$x = \frac{n_A}{n_A + n_B} = \frac{n_A}{n} \tag{3}$$

$$y = \frac{n_A{}^0 - n_A}{(n_A{}^0 - n_A) + (n_B{}^0 - n_B)} = \frac{n_A{}^0 - n_A}{n^0 - n} \tag{4}$$

The cut is given by

$$\theta = \frac{(n_A{}^0 - n_A) + (n_B{}^0 - n_A)}{n_A{}^0 + n_B{}^0} = \frac{n^0 - n}{n^0} \tag{5}$$

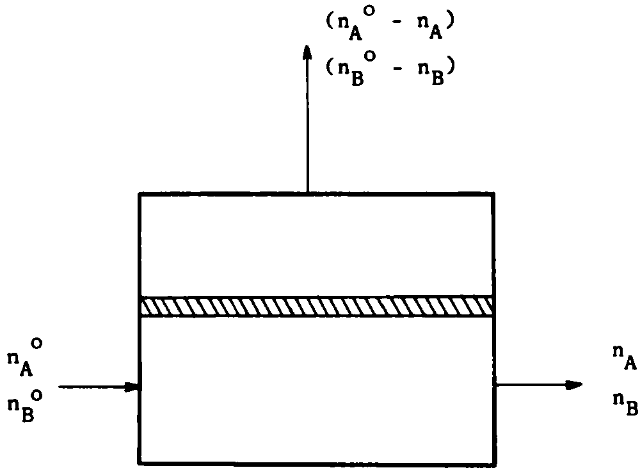

FIG. 1. Diffusion cell.

Combining the above equations one obtains

$$x^0 = \theta y + (1 - \theta)x \tag{6}$$

Then Eq. (1) becomes

$$\frac{nx}{n^0x^0} = \left[\frac{n(1 - x)}{n^0(1 - x^0)}\right]^\alpha \tag{7}$$

Using Eq. (5),

$$\frac{(1 - \theta)x}{x^0} = \left[\frac{(1 - \theta)(1 - x)}{(1 - x^0)}\right]^\alpha \tag{8}$$

This equation can be written as

$$\left(\frac{1 - x^0}{x^0}\right)\left(\frac{x}{1 - x}\right) = \left[\frac{(1 - \theta)(1 - x)}{(1 - x^0)}\right]^{\alpha - 1} \tag{9}$$

Raising to the $-\alpha/(\alpha - 1)$ power on both sides,

$$\left[\frac{(1 - x^0)}{(1 - \theta)(1 - x)}\right]^\alpha = \left(\frac{x^0}{1 - x^0}\right)^{\alpha/(\alpha-1)}\left(\frac{1 - x}{x}\right)^{\alpha/(\alpha-1)} \tag{10}$$

Combining Eqs. (8) and (10),

$$\frac{x^0}{(1 - \theta)x} = \left(\frac{x^0}{1 - x^0}\right)^{\alpha/(\alpha-1)}\left(\frac{1 - x}{x}\right)^{\alpha/(\alpha-1)} \tag{11}$$

Substituting Eq. (6) for x^0 into Eq. (11),

$$\frac{\theta y + (1-\theta)x}{(1-\theta)x} = \left[\frac{\theta y + (1-\theta)x}{1 - \theta y - (1-\theta)x}\right]^{\alpha/(\alpha-1)} \left(\frac{1-x}{x}\right)^{\alpha/(\alpha-1)} \quad (12)$$

Solving for y,

$$y = \left(\frac{1-\theta}{\theta}\right) x \left[\left(\frac{1-x}{x}\right)^{\alpha/(\alpha-1)} \left\{\frac{\theta y + (1-\theta)x}{1 - \theta y - (1-\theta)x}\right\}^{\alpha/(\alpha-1)} - 1\right]$$

$$= \left(\frac{1-\theta}{\theta}\right) x^{-1/(\alpha-1)} \left[(1-x)^{\alpha/(\alpha-1)} \times \left\{\frac{\theta y + (1-\theta)x}{1 - \theta y - (1-\theta)x}\right\}^{\alpha/(\alpha-1)} - x^{\alpha/(\alpha-1)}\right] \quad (13)$$

Finally, using the same notation as Naylor and Backer,

$$y = \left(\frac{1-\theta}{\theta}\right) x^{-1/\epsilon} \left[(1-x)^{\sigma} \left\{\frac{\theta y + (1-\theta)x}{1 - \theta y - (1-\theta)x}\right\}^{\sigma} - x^{\sigma}\right] \quad (14)$$

This is the same equation as given by Naylor and Backer (*2*).

It should be noted that the actual separation factor α becomes the ideal separation factor when the pressure of the undiffused stream becomes zero.

List of Symbols

n	moles per unit time
x	mole fraction of light component in depleted stream leaving a stage
y	mole fraction of light component in enriched stream

Greek Letters

α	separation factor
ϵ	$\epsilon = \alpha - 1$
θ	cut
σ	$\sigma = \alpha/\epsilon = \alpha/(\alpha - 1)$

Superscript

0	feed stream

Subscripts

A	light component
B	heavy component

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